3.4

$P(B|T) = P(T|B)\ P(B)/\ P(T)$.

$P(T) = P(T \mid B) * P(B) + P(T \mid B') * P(B')$

$0.1000061 = 0.7 * .000013 + 0.1 * (1-.000013)$

$P(B|T) = 0.7 * .000013 / .1000061 = .000090994$

$P(B|T') = P(T|B') * P(B') / P(T)$

$.000086994 = 0.1 * (1-.000013) / .1000061$

3.9

$P(A) = 3/4$ $P(B) = 2/5$ $P(A \cup B) = 4/5$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$7/20 = 3/4 + 2/5 - 4/5$

$P(A \cap B) = P(B \mid A) * P(A)$ and so

$P(B \mid A) = P(A) / P(A \cap B)$

$3/14 = (3/4) / (7/20)$

3.11

a. The probability that the driver's blood level does not exceed the legal limit given that the driver tested positive.

b. We are given that $P(A \mid B) = P(A' \mid B') = p = .95$

$P(B) = .05$, so $P(B') = 1.0 - .05 = 0.95$

Observe first that $(A \cap B')$ and $(A' \cap B')$ are disjoint, and that the union of these two sets is B'. It follows that $P(B') = P((A \cap B') \cup (A' \cap B')) = P(A \cap B') + P(A' \cap B')$
Rearranging terms gives

$P(A \cap B') = P(B') - P(A' \cap B')$

It follows that

$P(A \mid B')*P(B') = P(B') - P(A' \mid B') * P(B')$. So long as $P(B') > 0$, we can divide through by $P(B')$ giving

$P(A \mid B') = 1 - P(A' \mid B')$.

We wish to find $P(B' \mid A) = P(A \mid B')P(B') / P(A)$

$P(A) = P(A \mid B) * P(B) + P(A \mid B') * P(B')$

$P(A) = p * 0.05 + (1-p) * (1 - 0.05)$

Then $P(B' \mid A) = (1-p) * (1- 0.05) / (p * .05 + (1-p) * 1-.05)$

For $p = .95$, this gives

$(.05 * ..95) / (.95 * .05 + .05 * .95) = ½$

c. We want $P(B|A) = P(A|B) * P(B) / P(A)$

$= p * 0.5 / (p*.05 + (1-p) * .95) = .9$

Solving for p:

$.05p = .045p + .855 - .855p$
$.86p = .855$
$p = 99.42\%$

3.16
a:
$P(T|D) = .98$
$P(T'|D') = .95$
$P(D) = .01$
$P(D') = .99$

$P(D|T) = P(T|D) * P(D) / P(T)$

$P(T) = P(T|D)*P(D) + P(T|D') *P(D')$
$.0593 = 0.98 * .01 + (1-.95) * .99$

$P(D|T) = .98 * .01 / .0593 = .165$

b:
$P(D|(S\cap T)) = P(S\cap T|D) * P(D) / P(S \cap T)$

$P(S\cap T) = P(S\cap T|D) * P(D) + P(S\cap T)|D') * P(D')$
$=P(S)|D) *P(D) * P(T|D)*P(D) + P(S|D')*P(D')*P(T|D')*P(D)$
$=.98*.01*.98*.01 + .05*.99 *.05*.99 = .00254629b$

$P(D|(S\cap T) = .00009604/.00254629 = 0.28$

3.18
We are given that $0 < P(A) < 1$ and $0 < P(B) < 1$.

a) If A and B are disjoint, then $A\cap B = \Phi$ so $0 = P(A\cap B) = P(A)*P(B|A)$.
We are given that $P(A) > 0$. Thus $P(B|A)$ must be 0.
But $P(B) > 0$. Hence, $P(B|A) \neq P(B)$ and A and B are not independent.

b) If A and B are independent, then $P(A \cap B) = P(A) * P(B) > 0$ since both A and B are non-zero. But this means that $A \cap B$ is not empty (otherwise the probability of their intersection would be 0.) Hence, A and B are not disjoint.

c) Suppose A is a subset of B. Then $A \cap B = A$. Thus $P(A) = P(A \cap B) = P(A)*P(B|A)$ and we see that $P(B|A) = 1 \neq P(B)$ since $P(B) > 1$. Thus A and B are not independent.

d) A is a subset of A U B, and by part c) above cannot in any event be independent.